$dq/d\theta$ is close to θ_{cr_1}. This is confirmed by the data of work [5]: $\theta_m - \theta_{cr_1} \simeq 40 - 27 = 13°C$, $\theta_{cr_2} - \theta_m \simeq 150 - 40 = 110°C$.

3. The method suggested by Dr Stephan is known [6]. However the present author's [2] is still useful. First, at a horizontal plate the conditions are readily obtained when all the points of the surface are at strictly equal conditions and at equal temperature. Second, the procedure which uses condensing vapour as the heating medium is relatively simple and therefore is widely used for investigation of transient boiling heat transfer.

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NOTE ON THERMAL INSTABILITY OF A HORIZONTAL LAYER OF NON-NEWTONIAN FLUID HEATED FROM BELOW

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In a recent paper Tien *et al.* [1] considered the convective stability of a horizontal layer of a power-law fluid which is heated from below. They used an approximate theoretical analysis to derive conditions for stability. Apparently, these authors were not aware of the recent linearized stability analysis by Green [2] and by Vest and Arpaci [3]. Green used an Oldroyd constitutive equation and found that for fluids that are not extremely elastic the onset of motion occurs at the same Rayleigh number as for a Newtonian fluid. Vest and Arpaci obtained the same result for a Maxwell model. These results are reasonable since both models reduce to a Newtonian fluid in the limit of zero shear. The power-law model used by Tien *et al.* does not reduce to a Newtonian fluid in the limit of zero shear and thus is not the best choice of constitutive equation to study the onset of convective motion.

The stability analysis used by Tien *et al.* is closely related to the energy method of stability analysis and should be useful with other constitutive equations. The energy method was first applied to convective stability problems by Joseph [4] and to the stability of non-Newtonian fluids by Feinberg [5]. This method is closely related to Liapunov stability methods [6]. Since the development of the stability equations is considered in detail elsewhere [4–6], only a brief outline will be given.

The equations of motion and heat transfer are written for a basic flow $\boldsymbol{V}$, $\bar{\tau}$, $\bar{T}$, $\boldsymbol{p}$ and a disturbed flow $\boldsymbol{V}^*$, $\bar{\tau}^*$, T^*, p^*. The Boussinesq approximation is assumed. Define the perturbations,

$$\boldsymbol{u} = \boldsymbol{V}^* - \boldsymbol{V}, \qquad \Delta\tau = \tau^* - \bar{\tau}, \qquad \theta = T^* - \bar{T} \quad (1\text{A, B, C})$$

and subtract the basic flow from the disturbed flow. After forming the scalar product of $\boldsymbol{u}$ with the equations of motion and of θ with the heat-transfer equation, integrate the results over the flow volume V. This volume extends from $z = 0$ to $z = d$ and over a period of the perturbations in the x and y directions. The result is [4, 5]

$$\frac{dK}{dt} = \int_V \left[\frac{1}{\rho}\boldsymbol{u}\cdot\nabla\cdot\Delta\tau - \boldsymbol{u}\cdot\boldsymbol{D}\cdot\boldsymbol{u} - \alpha\theta\boldsymbol{g}\cdot\boldsymbol{u}\right] \times dV + \int_S \frac{p^* - \bar{p}}{\rho}\boldsymbol{u}\cdot\boldsymbol{N}\,dS \qquad (2)$$

$$\frac{\mathrm{d}\Theta}{\mathrm{d}t} = \int_V \left[-\theta \boldsymbol{u}\cdot\nabla\bar{T} + \kappa\theta\nabla^2\theta \right] \mathrm{d}V \tag{3}$$

where

$$K = \int_V \tfrac{1}{2}\boldsymbol{u}\cdot\boldsymbol{u}\,\mathrm{d}V, \qquad \Theta = \int_V \tfrac{1}{2}\theta^2\,\mathrm{d}V,$$

$$\boldsymbol{D} = \tfrac{1}{2}\left[\nabla\boldsymbol{V} + (\nabla\boldsymbol{V})^T\right] \tag{4A, B, C}$$

and S is the bounding surface of V, and N is the outward normal. The surface integral in equation (2) vanishes when the perturbations are periodic in the x and y directions and when $\mathbf{u}.\ \boldsymbol{N} = 0$ at $z = 0, d$. The convective system must be stable if $\mathrm{d}K/\mathrm{d}t < 0$ and $\mathrm{d}\Theta/\mathrm{d}t < 0$ as $t \to \infty$ [4–6]. Only the neutral case where $\mathrm{d}K/\mathrm{d}t = \mathrm{d}\Theta/\mathrm{d}t = 0$ will be considered.

For the basic system of a quiescent fluid layer with a linear temperature gradient β we have,

$$\boldsymbol{D} = 0, \qquad T = -\beta\boldsymbol{k}, \qquad \boldsymbol{g}\cdot\boldsymbol{u} = -g\boldsymbol{k}\cdot\boldsymbol{u}. \tag{5A, B, C}$$

For the neutral case equations (2) and (3) simplify to,

$$0 = \int_V \left(\frac{1}{\rho}\,\boldsymbol{u}\cdot\nabla\cdot\Delta\tau + \alpha\theta g\boldsymbol{k}\cdot\boldsymbol{u}\right)\mathrm{d}V \tag{6}$$

$$0 = \int_V (\theta\beta\boldsymbol{k}\cdot\boldsymbol{u} + \kappa\theta\nabla^2\theta)\,\mathrm{d}V. \tag{7}$$

Equations (6) and (7) are combined to obtain,

$$0 = \int_V \left(\frac{\beta}{\rho}\,\boldsymbol{u}\cdot\nabla\cdot\Delta\tau - \alpha g\kappa\theta\nabla^2\theta\right)\mathrm{d}V. \tag{8}$$

If the power-law form of constitutive equation is used for τ and all terms are made dimensionless following the method of Tien *et al.* [1], equation (8) reduces to equation (9) of Tien *et al.* (after correction of a typographical error in their equation). Any desired constitutive equation for τ could be used in equation (8). Joseph [4] and Feinberg [5] show how the volume integrals in equation (8) can be put into simpler form.

Rigorous methods for finding the conditions for which $\mathrm{d}K/\mathrm{d}t < 0$ and $\mathrm{d}\Theta/\mathrm{d}t < 0$ are discussed in detail elsewhere [4–6]. The approximate method of Tien *et al.* [1] should be useful for realistic constitutive equations, but is inconsistent with their use of the power-law model. For a constitutive equation which reduces to a Newtonian fluid in the limit of zero shear the use of the Newtonian solutions for marginal temperature and velocity fields would be an excellent approximation. Unfortunately, the power-law model does not reduce to a Newtonian fluid in the limit of zero shear.

Tien *et al.* were unable to explain why the two-dimensional roll gave a lower critical Rayleigh number than the hexagonal cell. A possible explanation for this result may be contained in the non-linear analysis of Segal [7]. For Newtonian fluids Segal showed that roll cells become stable instead of hexagonal cells as the Rayleigh number is increased above the critical value.

The experimental work reported by Tien *et al.* is apparently the first experimental data on convective stability of non-Newtonian fluids. It would be of interest if these authors characterized their fluids in terms of the Oldroyd and Maxwell constitutive equations so the theoretical results of Green [2] and Vest and Arpaci [3] could be checked.

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